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Fundamental strings and NS5-branes from unstable D-branes in supergravity

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Abstract

By using the non-supersymmetric p-brane solutions delocalized in arbitrary number of transverse directions in type II supergravities, we show how they can be regarded as interpolating solutions between unstable Dp-branes (a non-BPS D-brane or a pair of coincident D-brane-antiD-brane) and fundamental strings and also between unstable Dp-branes and NS5-branes. We also show that some of these solutions can be regarded as interpolating solutions between NS5/$\overline{\rm NS}$5 and Dp-branes (for $p \leq 5$). This gives a closed string description of the tachyon condensation and lends support to the conjecture that the open string theory on unstable D-branes at the tachyonic vacuum has soliton solutions describing not only the lower dimensional BPS D-branes, but also the fundamental strings as well as the NS5-branes.

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A non-BPS D-brane or a pair of coincident brane-antibrane system in type II string theories is unstable which is manifested by the presence of open string tachyon on their respective world-volumes [1, 2]. Because of this, these unstable D-branes decay and the decay occurs through a process called tachyon condensation. The tachyon condensation has been understood from the open string point of view both in the boundary CFT approach [3, 4, 5, 2] and the string field theory approach [6, 7]. In particular, it has been conjectured that the tachyonic vacuum in these theories describes the closed string vacuum without any brane and the various solitonic solutions representing the lower dimensional BPS D-branes, the fundamental strings as well as the Neveu-Schwarz 5-branes [8]. This is what is expected and evidences have been given for the lower dimensional BPS D-branes in [9] (the appearance of these objects can also be understood from K-theory [10]) and for the fundamental strings in [8, 11] from the open string point of view.

The appearance of the various decay products at the tachyonic vacuum from the Dp-$\overline{\text{D}}p$ (we denote an anti-brane with a bar) or from a non-BPS D$(p-1)$-brane can be understood as follows [11, 12]. For definiteness we will discuss the case of Dp-$\overline{\text{D}}p$-brane and briefly mention about non-BPS D$(p-1)$-branes. The world-volume theory on a pair of Dp-$\overline{\text{D}}p$ brane is a gauge theory with gauge group $U(1)\times U(1)$ and has a complex tachyon of charge $(1, -1)$. (For non-BPS D$(p-1)$-brane the gauge group is $Z_2 \times U(1)$ with a chargeless, real tachyon. The removal of $U(1)$ gauge degrees of freedom here can be achieved through confinement which can be discussed similarly as for the case of Dp-$\overline{\text{D}}p$-brane.) As the tachyon condenses, its phase can acquire a non-trivial winding number on a finite energy vortex solution of this theory which can be identified as BPS D$(p-2)$-brane. In this process the relative $U(1)$ gauge group is broken and the relative gauge field $(A_1 - A_2)$ gets removed by the Higgs mechanism. The overall gauge field $(A_1 + A_2)$ under which the tachyon is neutral gets also removed through confinement, which can also be understood by the dual Higgs mechanism. The only remnant of the gauge field is the confined electric flux string which can be identified as the fundamental string [11, 12]. This phenomenon can be understood by open D$(p-2)$-branes stretched between Dp-$\overline{\text{D}}p$-brane. The open D$(p-2)$-brane will induce a $(p-3)$ dimensional tachyonic object charged under the relative two $(p-2)$-forms $(A_1^{[p-2]} - A_2^{[p-2]})$ on the world-volume of Dp-$\overline{\text{D}}p$-brane. The dual of the corresponding field-strength is related with the electric flux associated with the overall gauge field $(A_1 + A_2)$. So, the $(p-3)$-dimensional tachyonic object is magnetically charged under (A_1+A_2) and after its condensation the overall $U(1)$ is removed by the dual Higgs mechanism [11] (a different mechanism, as opposed to $p > 2$, has been suggested [12] for $p \leq 2$ cases) giving a confined electric flux string identified as F-string.

We now come to NS5-brane. The NS5-$\overline{\text{NS}}$5 decaying into Dp branes with $p \leq 5$ can

be deduced either from the result NS5-$\overline{\text{NS}}$5 decaying into D2 [11] and by applying T-dualities subsequently along the NS5 directions or from the D5-$\overline{\text{D}}$5 decaying into D3 and by applying S-duality and subsequent T-dualities. On the other hand, Dp-$\overline{\text{D}}p$ decaying into NS5 is less obvious but can still be deduced partially from the above. We mentioned NS5-$\overline{\text{NS}}$5 decays into D5 and S-dual of this gives D5-$\overline{\text{D}}$5 decaying into NS5. Applying T-dualities along the NS5 directions gives Dp-$\overline{\text{D}}p$ decaying into NS5 for $p \leq 5$. Now in order to see the process D6-$\overline{\text{D}}$6 decaying into NS5, we can start with D0-$\overline{\text{D}}$0 decaying into F-string [12] and apply T- and S-duality successively on it. To be precise, if the F-string above lies along x^1-direction, then the application of ($\text{T}_1\text{ST}_{34}\text{ST}_{56}\text{ST}_{234}$) from the right on both sides of D0-$\overline{\text{D}}$0 decaying into F, where the subscript in T refers to application of T-duality along those directions and S refers to S-duality, will give D6-$\overline{\text{D}}$6 decaying into NS5. Some relevant discussions can also be found in [13, 14].

In this paper we will give a closed string or supergravity description of the appearance of F-strings and NS5-branes from Dp-$\overline{\text{D}}p$-brane or non-BPS Dp-brane described above. It should be pointed out that in the supergravity picture there is no explicit tachyon field, but as it was argued in [15, 16] tachyon vev can appear as parameters labelling the supergravity solution. Indeed the non-susy p-brane solutions [17, 18] which in some special cases can be argued to represent Dp-$\overline{\text{D}}p$-brane system or non-BPS Dp-brane contain some parameters. By using their supersymmetric reduction we have argued in [16], how these parameters are related to the microscopic physical parameters as well as the tachyon vev to correctly describe the tachyon condensation process. In a different approach we have shown earlier [19], how by delocalizing the non-susy Dp-brane solution in one transverse direction, we can regard it (a) as an interpolating solution between non-BPS D$(p+1)$ brane and a BPS Dp-brane similar to the picture of tachyon condensation on the kink solution [2] and (b) as an interpolating solution between non-BPS D$(p+1)$-brane and the supergravity description of tachyon matter [20] similar to the rolling tachyon picture [21] in the open string description. For case (a) the delocalized direction was space-like and for (b) the delocalized direction was time-like. This result gave us confidence to believe that the supergravity solution indeed correctly represent the Dp-$\overline{\text{D}}p$-brane system or non-BPS Dp-brane. We have further shown in [22], by delocalizing the non-susy Dp-brane to two spatial transverse directions, how this solution can be regarded as an interpolating solution between D$(p+2)$-$\overline{\text{D}}(p+2)$, non-BPS D$(p+1)$ and BPS Dp-brane giving the complete descent relation of Sen [2] under tachyonic kink and vortex solutions. Since we have already described how the lower dimensional D-branes can be obtained from Dp-$\overline{\text{D}}p$ and non-BPS Dp in our earlier works, in this paper we will show how the other closed string objects like F-strings and NS5-branes can be obtained from Dp-$\overline{\text{D}}p$ (or non-BPS

Dp). We will also show how Dp-branes can be obtained from NS5-$\overline{\text{NS}}$5. This gives a closed string description of tachyon condensation on unstable D-brane and gives more evidence to the conjecture that tachyonic vacuum is a closed string vacuum consisting not only of lower dimensional D-branes, but also of F-strings and NS5-branes.

In order to show various processes described above from supergravity we first construct the non-supersymmetric p-brane solutions delocalized in q spatial directions in arbitrary space-time dimensions d. The supergravity action we consider is,

$$S = \int d^d x \sqrt{-g} \left[R - \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2\cdot(d-p-2)!} e^{a\phi} F^2_{[d-p-2]} \right] \tag{1}$$

where $g_{\mu\nu}$, with $\mu, \nu = 0, 1, \ldots, d-1$, is the metric and $g = \det(g_{\mu\nu})$, R is the scalar curvature, ϕ is the dilaton, $F_{[d-p-2]}$ is the field strength of a $(d-p-3)$-form gauge field and a is the dilaton coupling. The action (1) represents the bosonic sector of the low energy effective action of string/M theory dimensionally reduced to d-dimensions. Now in order to obtain the delocalized solutions in q transverse directions, we have to solve the equations of motion from (1) with the following ansatz for the metric and the $(d-p-2)$-form field strength,

$$\begin{aligned} ds^2 &= e^{2A(r)} \left(dr^2 + r^2 d\Omega^2_{d-p-q-2} \right) + e^{2B(r)} \left(-dt^2 + \sum_{i=1}^{p} dx_i^2 \right) + \sum_{i=2}^{q+1} e^{2C_{i-1}(r)} dx^2_{p+i-1} \\ F_{[d-p-2]} &= b \ \mathrm{Vol}(\Omega_{d-p-q-2}) \wedge dx_{p+1} \ldots \wedge dx_{p+q} \end{aligned} \tag{2}$$

In the above $r = (x^2_{p+q+1} + \cdots + x^2_{d-1})^{1/2}$, $d\Omega^2_{d-p-q-2}$ is the line element of a unit $(d-p-q-2)$-dimensional sphere, $\mathrm{Vol}(\Omega_{d-p-q-2})$ is its volume-form and b is the magnetic charge parameter. The solutions (2) represent magnetically charged p-brane solutions delocalized in transverse $x_{p+1}, x_{p+2} \ldots, x_{p+q}$, directions. The equations of motion will be solved with the following gauge condition,

$$(p+1)B(r) + (d-p-q-2)A(r) + \sum_{i=2}^{q+1} C_{i-1}(r) = \ln G(r) \tag{3}$$

Note that as $G(r) \to 1$, the above condition reduces to the extremality or the supersymmetry condition. As mentioned in [18], the consistency of the equations of motion dictates that the non-extremality function $G(r)$ can take three different forms and we will need only one of them for our purpose which is,

$$G(r) = 1 - \frac{\omega^{2(d-p-q-3)}}{r^{2(d-p-q-3)}} = \left(1 + \frac{\omega^{d-p-q-3}}{r^{d-p-q-3}}\right)\left(1 - \frac{\omega^{d-p-q-3}}{r^{d-p-q-3}}\right) = H(r)\tilde{H}(r) \tag{4}$$

By solving the equations of motion following from (1), we can obtain all the functions $A(r)$, $B(r)$ and $C_1(r),\ldots,C_q(r)$ appearing in the metric subject to some constraints. The solutions therefore take the forms,

$$
\begin{aligned}
ds^2 &= F^{\frac{4(p+1)}{(d-p-3)\chi}}(H\tilde{H})^{\frac{2}{d-p-q-3}}\left(\frac{H}{\tilde{H}}\right)^{\frac{-2\sum_{i=2}^{q+1}\delta_i}{d-p-q-3}}\left(dr^2+r^2 d\Omega^2_{d-p-q-2}\right)\\
&\quad + F^{-\frac{4}{\chi}}\left(-dt^2+\sum_{i=1}^{p}dx_i^2\right)+F^{\frac{4(p+1)}{(d-p-3)\chi}}\sum_{i=2}^{q+1}\left(\frac{H}{\tilde{H}}\right)^{2\delta_i}dx^2_{p+i-1}\\
e^{2\phi} &= F^{-\frac{4a(d-2)}{(d-p-3)\chi}}\left(\frac{H}{\tilde{H}}\right)^{2\delta_1}, \qquad F_{[d-p-2]} = b\ \mathrm{Vol}(\Omega_{d-p-q-2})\wedge dx_{p+1}\ldots\wedge dx_{p+q}
\end{aligned}
\tag{5}
$$

These are magnetically charged, non-supersymmetric p-brane solutions delocalized in q directions and the corresponding electrically charged solutions can be obtained by simply replacing $F \to e^{-a\phi} * F$. The field-strength for the electrical solution takes the form (which follows from (5))

$$
F_{[p+2]} = \frac{\sinh 2\theta}{2} d\left(\frac{C}{F}\right)\wedge dt \wedge dx_1 \ldots \wedge dx_p \tag{6}
$$

The functions F and C appearing in the above solutions are defined as

$$
\begin{aligned}
F &= \cosh^2\theta\left(\frac{H}{\tilde{H}}\right)^{\alpha}-\sinh^2\theta\left(\frac{\tilde{H}}{H}\right)^{\beta}\\
C &= \left(\frac{H}{\tilde{H}}\right)^{\alpha}-\left(\frac{\tilde{H}}{H}\right)^{\beta}
\end{aligned}
\tag{7}
$$

In the solutions (5) and (6) above there are $(q+5)$ integration constants and they are ω, θ, α, β, $\delta_1,\ldots,\delta_{q+1}$. b is the charge parameter. However, not all the parameters are independent. From consistency of the equations of motion we obtain three relations among the parameters and they are

$$
\alpha-\beta = a\delta_1 \tag{8}
$$

$$
\frac{1}{2}\delta_1^2+\frac{2\alpha(\alpha-a\delta_1)(d-2)}{\chi(d-p-3)}+\frac{2\sum_{i<j=2}^{q+1}\delta_i\delta_j}{d-p-q-3} = \left(1-\sum_{i=2}^{q+1}\delta_i^2\right)\frac{d-p-q-2}{d-p-q-3} \tag{9}
$$

$$
b = \sqrt{\frac{4(d-2)}{(d-p-3)\chi}}(d-p-q-3)\omega^{d-p-q-3}(\alpha+\beta)\sinh 2\theta \tag{10}
$$

where χ in the above is defined as $\chi = 2(p+1)+a^2(d-2)/(d-p-3)$ and the dilaton coupling is given by $a^2 = 4-2(p+1)(d-p-3)/(d-2)$ for the supergravities with maximal

supersymmetry (the case we are considering). So, the number of independent parameters in the above solutions is $(q+3)$.

For our purpose of showing the appearance of F-strings from the Dp-$\overline{\rm D}p$ system we here write the non-susy F-string solutions delocalized in $(p-1)$ transverse directions from the general solutions (5) and (6) by putting $d=10$, $p=1$, $q=p-1$, $a=1$ (which implies $\chi=16/3$) as,

$$\begin{aligned} ds^2 &= F^{\frac{1}{4}}(H\tilde{H})^{\frac{2}{7-p}}\left(\frac{H}{\tilde{H}}\right)^{-(2\sum_{i=2}^{p}\delta_i)/(7-p)}\left(dr^2+r^2 d\Omega_{8-p}^2\right) \\ & \quad + F^{-\frac{3}{4}}\left(-dt^2+dx_1^2\right)+F^{\frac{1}{4}}\sum_{i=2}^{p}\left(\frac{H}{\tilde{H}}\right)^{2\delta_i}dx_i^2 \\ e^{2\phi} &= F^{-1}\left(\frac{H}{\tilde{H}}\right)^{2\delta_1}, \quad B^{(2)} = \frac{\sinh 2\theta}{2}\left(\frac{C}{F}\right)dt\wedge dx_1 \end{aligned} \tag{11}$$

with the parameter relation

$$\frac{1}{2}\delta_1^2+\frac{1}{2}\alpha(\alpha-\delta_1)+\frac{2\sum_{i<j=2}^{p}\delta_i\delta_j}{7-p} = \left(1-\sum_{i=2}^{p}\delta_i^2\right)\frac{8-p}{7-p} \tag{12}$$

The functions F and C appearing in (11) are as given in eq.(7). Also here $H=1+\omega^{7-p}/r^{7-p}$ and $\tilde{H}=1-\omega^{7-p}/r^{7-p}$. There are $(4+p)$ parameters in the solution and they are α, β, ω, θ, δ_1, $\ldots$, δ_p, however, there are two relations among them, one is given in (12) and the other is $\alpha-\beta=\delta_1$. So eliminating α, β we have a $(2+p)$ parameter solution in (11). It is clear from (11) that the solution can be made localized if the coefficients of all the terms $dx_2^2,\ldots,dx_p^2$ match with that of the term $(-dt^2+dx_1^2)$. This is indeed possible if $\theta=0$, so that $F=\left(\frac{H}{\tilde{H}}\right)^{\alpha}$ and we set $2\delta_i=-\alpha$, for, $i=2,\ldots,p$. If we now define new parameters as,

$$\tilde{\alpha}=\frac{6}{7-p}\alpha, \qquad \tilde{\delta}_1=\delta_1+\frac{2(p-4)}{7-p}\alpha \tag{13}$$

Then the solution (11) reduces to,

$$\begin{aligned} ds^2 &= (H\tilde{H})^{\frac{2}{7-p}}\left(\frac{H}{\tilde{H}}\right)^{\frac{p+1}{8}\tilde{\alpha}}\left(dr^2+r^2 d\Omega_{8-p}^2\right)+\left(\frac{H}{\tilde{H}}\right)^{-\frac{7-p}{8}\tilde{\alpha}}\left(-dt^2+\sum_{i=1}^{p}dx_i^2\right) \\ e^{2\phi} &= \left(\frac{H}{\tilde{H}}\right)^{-a\tilde{\alpha}+2\tilde{\delta}_1}, \quad B^{(2)}=0, \quad a=\frac{p-3}{2} \end{aligned} \tag{14}$$

and the parameter relation (12) reduces to,

$$\frac{1}{2}\tilde{\delta}_1^2+\frac{1}{2}\tilde{\alpha}(\tilde{\alpha}-a\tilde{\delta}_1)=\frac{8-p}{7-p} \tag{15}$$

We recognize the solutions (14) and (15) as representing either the chargeless Dp-$\overline{\mathrm{D}}p$ solution or the non-BPS Dp solution [18] depending on whether p is even or odd and whether we are considering type IIA or type IIB theory. We would like to point out that the interpretations for the chargeless solutions (as in eq.(14) or even the cases discussed below in eqs.(18) and (21)) are not unique. This indicates that there are more decay processes than those considered in this paper and will be discussed elsewhere.

Now in order to see how the solution (11) reduces to F-string solution we take the following scaling limit

$$\omega^{7-p} = \epsilon \bar{\omega}^{7-p}, \qquad (\alpha + \beta) \sinh 2\theta = \epsilon^{-1} \tag{16}$$

where the parameter $\epsilon \to 0$ and $\bar{\omega} =$ fixed. Using (16) we find from (7) $F \to \bar{H} = 1 + \frac{\bar{\omega}^{7-p}}{r^{7-p}}$ and H, $\tilde{H} \to 1$. The solution (11) then reduces to

$$\begin{aligned} ds^2 &= \bar{H}^{\frac{1}{4}} \left(dr^2 + r^2 d\Omega_{8-p}^2 + \sum_{i=2}^{p} dx_i^2 \right) + \bar{H}^{-\frac{3}{4}} \left(-dt^2 + dx_1^2 \right) \\ e^{2\phi} &= \bar{H}^{-1}, \qquad B^{(2)} = \left(1 - \bar{H}^{-1} \right) dt \wedge dx_1 \end{aligned} \tag{17}$$

We recognize (17) to be the fundamental string solution in Einstein frame delocalized in $(p-1)$ directions. However, since this is a BPS solution, we can always make the string solution localized by replacing $(p-1)$-dimensional source by a point source. The harmonic function $\bar{H} = 1 + \frac{\bar{\omega}^{7-p}}{r^{7-p}}$ will then be replaced by $\bar{H} = 1 + \frac{\bar{\omega}^6}{r^6}$. The solution (17) will reduce to a localized F-string solution. (Note that this localization is not a smooth operation in the parameter space. However, for the supergravity BPS solution this is a familiar procedure when we take T-duality [23] on it. A similar situation arises also in the open string picture [12] and to get fully localized F-string from the decay of D2-$\overline{\mathrm{D}}$2 a non-pertubative technique was used there.) This shows that the solution (11) interpolates between Dp-$\overline{\mathrm{D}}p$-brane (or a non-BPS Dp-brane) solution and the fundamental string solution. We remark that the fundamental string solutions in both type IIA and type IIB string theory have the same forms. So, if we are in type IIA (IIB) theory then the solution (11) interpolates between Dp-$\overline{\mathrm{D}}p$ (non-BPS Dp) and F-string for $p =$ even; and it interpolates between non-BPS Dp (Dp-$\overline{\mathrm{D}}p$) and F-string for $p =$ odd. We thus see that as we move in the parameter space the interpolation has the similar effect as the open string tachyon condensation.

Now we will see how NS5-branes can be obtained from Dp-$\overline{\mathrm{D}}p$ solution or non-BPS Dp solution. We restrict our discussion for $p = 5$ and 6 only and comment on $p < 5$ later. In order to show this we start with the non-susy NS5-brane solution delocalized in

p' directions. The solution in this case can be written from (5) by putting $d = 10$, $p = 5$, $q = p'$, $a = -1$ (which implies $\chi = 16$) as,

$$
\begin{aligned}
ds^2 &= F^{\frac{3}{4}}(H\tilde{H})^{\frac{2}{2-p'}}\left(\frac{H}{\tilde{H}}\right)^{-\frac{(2\sum_{i=2}^{p'+1}\delta_i)}{(2-p')}}\left(dr^2 + r^2 d\Omega_{3-p'}^2\right) \\
&\quad + F^{-\frac{1}{4}}\left(-dt^2 + \sum_{j=1}^{5} dx_j^2\right) + F^{\frac{3}{4}}\sum_{i=2}^{p'+1}\left(\frac{H}{\tilde{H}}\right)^{2\delta_i} dx_{4+i}^2 \\
e^{2\phi} &= F\left(\frac{H}{\tilde{H}}\right)^{2\delta_1}, \qquad F_{[3]} = b\mathrm{Vol}(\Omega_{3-p'}) \wedge dx_6 \wedge \ldots \wedge dx_{5+p'}
\end{aligned}
\tag{18}
$$

Where the function F is as given in (7) with $H = 1 + \frac{\omega^{2-p'}}{r^{2-p'}}$ and $\tilde{H} = 1 - \frac{\omega^{2-p'}}{r^{2-p'}}$. We note that p' can be 0 or 1. When it is zero the solution (18) represents localized non-susy NS5-brane and when it is 1 it represents non-susy NS5-brane delocalized in one direction. The parameters in the above solution are related as, $\alpha - \beta = -\delta_1$, $b = (2-p')(\alpha+\beta)\omega^{2-p'}\sinh 2\theta$ and

$$
\frac{1}{2}\delta_1^2 + \frac{1}{2}\alpha(\alpha + \delta_1) + \frac{2\sum_{i<j=2}^{p'+1}\delta_i\delta_j}{2-p'} = \left(1 - \sum_{i=2}^{p'+1}\delta_i^2\right)\frac{3-p'}{2-p'}
\tag{19}
$$

Note that the last term in the lhs of (19) is redundant since it is zero for both $p' = 0, 1$.

Let us discuss $p' = 0$ first. We note from (18) that if we put $\theta = b = 0$ and define new parameters in terms of the old parameters as $\tilde{\alpha} = \alpha$, $\tilde{\delta}_1 = \alpha + \delta_1$, then the solution reduces to that of a D5-$\overline{\text{D}}$5-solution or non-BPS D5 solution. Even the parameter relation (19) reduces to $(1/2)\tilde{\delta}_1^2 + (1/2)\tilde{\alpha}(\tilde{\alpha} - \tilde{\delta}_1) = 3/2$ i.e. that of a D5-$\overline{\text{D}}$5 or non-BPS D5 solution. In order to get BPS NS5 brane solution from (18) we simply scale $\omega^2 = \epsilon\,\bar{\omega}^2$ and $(\alpha+\beta)\sinh 2\theta = \epsilon^{-1}$, where $\epsilon \to 0$ and $\bar{\omega}$ is fixed. Then F reduces to $\bar{H} = 1 + \frac{\bar{\omega}^2}{r^2}$ and H, $\tilde{H}$ goes to 1 and we recover BPS NS5-brane solution from (18). The form of the NSNS field in this case is $F_{[3]} = b\mathrm{Vol}(\Omega_3)$. We have thus seen that the solution (18) for $p' = 0$ can be regarded as interpolating solution between the chargeless D5-$\overline{\text{D}}$5 (or non-BPS D5) solution and BPS NS5-brane solution.

On the other hand for $p' = 1$, the solution (18) can be made localized if $\theta = b = 0$ and $\delta_2 = -\alpha/2$. Now defining new parameters in terms of old parameters as $\tilde{\alpha} = 2\alpha$ and $\tilde{\delta}_1 = 2\alpha + \delta_1$ we can rewrite the solution (18) as

$$
\begin{aligned}
ds^2 &= (H\tilde{H})^2\left(\frac{H}{\tilde{H}}\right)^{\frac{7}{8}\tilde{\alpha}}\left(dr^2 + r^2 d\Omega_2^2\right) + \left(\frac{H}{\tilde{H}}\right)^{-\frac{1}{8}\tilde{\alpha}}\left(-dt^2 + \sum_{i=1}^{6} dx_i^2\right) \\
e^{2\phi} &= \left(\frac{H}{\tilde{H}}\right)^{-\frac{3}{2}\tilde{\alpha} + 2\tilde{\delta}_1}, \qquad F_{[3]} = 0
\end{aligned}
\tag{20}
$$

The parameter relation (19) can also be rewritten as $(1/2)\tilde{\delta}_1^2 + (1/2)\tilde{\alpha}(\tilde{\alpha} - (3/2)\tilde{\delta}_1) = 2$. The above solution represents precisely the D6-$\overline{\rm D}$6 (or non-BPS D6) brane solution. Again in order to get BPS NS5-brane solution from (18), we do the same scaling of ω and θ as before. But, since in this case the harmonic functions are that of a 6-brane involving $(1/r)$ and not $(1/r^2)$, we get a delocalized NS5-brane where the form-field is given as $F_{[3]} = b{\rm Vol}(\Omega_2) \wedge dx_6$. But since this solution is BPS, one can make it localized as mentioned before. This shows that the solution (18) for $p' = 1$ can indeed be regarded as interpolating solution between D6-$\overline{\rm D}$6 (or non-BPS D6) brane solution and BPS NS5-brane solution.

For other values of $p < 5$, it is not clear how the above procedure will work. But one can start with the non-isotropic, non-susy NS5-brane solution and try to make it Dp-$\overline{\rm D}p$ (or non BPS Dp) solution as we have done above. But it can be easily checked that the resulting solution can not be made localized Dp-$\overline{\rm D}p$ (or non-BPS Dp) by only adjusting the parameters. If we look at only the linear part of the solution then the solution will look like localized solution. The meaning of this is not entirely clear to us.

Next we will show how Dp-branes (for $p \leq 5$) can be obtained as the decay product of NS5-$\overline{\rm NS}$5. In supergravity we describe the process with a solution which interpolates between NS5-$\overline{\rm NS}$5 and Dp brane solutions. For this purpose we write non-susy Dp-brane solution delocalized in $(5-p)$-directions from (5) and (6) by putting $d = 10$, $p = p$, $q = 5-p$, $a = (p-3)/2$ (which implies $\chi = 32/(7-p)$) as,

$$\begin{aligned}
ds^2 &= F^{\frac{p+1}{8}}(H\tilde{H})\left(\frac{H}{\tilde{H}}\right)^{-\sum_{i=2}^{6-p}\delta_i}\left(dr^2 + r^2 d\Omega_3^2\right)\\
&\quad + F^{-\frac{7-p}{8}}\left(-dt^2 + \sum_{i=1}^{p} dx_i^2\right) + F^{\frac{p+1}{8}}\sum_{i=2}^{6-p}\left(\frac{H}{\tilde{H}}\right)^{2\delta_i} dx_{i+p-1}^2\\
e^{2\phi} &= F^{-\frac{p-3}{2}}\left(\frac{H}{\tilde{H}}\right)^{2\delta_1}\\
A^{(p+1)} &= \frac{\sinh 2\theta}{2}\left(\frac{C}{F}\right) dt \wedge dx_1 \wedge \ldots \wedge dx_p
\end{aligned} \tag{21}$$

The parameters satisfy,

$$\frac{1}{2}\delta_1^2 + \frac{1}{2}\alpha\left(\alpha - a\delta_1\right) + \sum_{i<j=2}^{6-p}\delta_i\delta_j = \left(1 - \sum_{i=2}^{6-p}\delta_i^2\right)\frac{3}{2} \tag{22}$$

Where in the above the functions F, C are as defined before, but H, $\tilde{H}$ have the forms $H = 1 + \frac{\omega^2}{r^2}$ and $\tilde{H} = 1 - \frac{\omega^2}{r^2}$. The above solution can be made localized if we put $\theta = 0$ and $-2\delta_i = \alpha$, for $i = 2, \ldots, (6-p)$. Defining new parameters as $\tilde{\alpha} = (7-p)\alpha/2$ and

$\tilde{\delta}_1 = \delta_1 - \alpha$, the solution (21) reduces to,

$$
\begin{aligned}
ds^2 &= (H\tilde{H})\left(\frac{H}{\tilde{H}}\right)^{\frac{3}{4}\tilde{\alpha}}\left(dr^2 + r^2 d\Omega_3^2\right) + \left(\frac{H}{\tilde{H}}\right)^{-\frac{1}{4}\tilde{\alpha}}\left(-dt^2 + \sum_{i=1}^{5} dx_i^2\right) \\
e^{2\phi} &= \left(\frac{H}{\tilde{H}}\right)^{\tilde{\alpha}+2\tilde{\delta}_1}, \qquad A^{(p+1)} = 0
\end{aligned}
\tag{23}
$$

and the parameter relation (22) takes the form $(1/2)\tilde{\delta}_1^2 + (1/2)\tilde{\alpha}(\tilde{\alpha} + \tilde{\delta}_1) = 3/2$. This is precisely the chargeless NS5-$\overline{\text{NS}}$5 brane solution [18]. Now in order to see how the solution (21) reduces to the BPS Dp-brane solution, we scale the parameters ω and θ exactly as before and send H, $\tilde{H} \to 1$ and $F \to \bar{H} = 1 + \frac{\bar{\omega}^2}{r^2}$. $A^{(p+1)}$ in (21) reduces to $(1-\bar{H}^{-1})dt \wedge dx_1 \wedge \ldots \wedge dx_p$. The solution then reduces precisely to the Dp-brane solution delocalized in $(5-p)$ directions. However, it can be easily localized by replacing the $(5-p)$-dimensional source by a point source as we mentioned earlier. This therefore shows that the delocalized solution (21) can indeed be regarded as the interpolating solution between the chargeless NS5-$\overline{\text{NS}}$5-brane and the Dp-brane. From the world-volume point of view this interpolation means Dp-branes appear as a decay product of NS5-$\overline{\text{NS}}$5-brane.

We have thus seen how the various delocalized, non-susy p-brane solutions of type II supergravities can be regarded as interpolating solutions between Dp-$\overline{\text{D}}p$ (or non-BPS Dp)-brane and F-string, between Dp-$\overline{\text{D}}p$ (or non-BPS Dp)-brane and NS5-brane and NS5-$\overline{\text{NS}}$5-brane and Dp-brane solutions, by adjusting and scaling the parameters of the solutions. The open string description of some of these processes were well-understood in terms of tachyon condensation. We here obtain a closed string or supergravity picture of these processes.

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